Comparison of chain event graphs and Bayesian networks for the Monty Hall problem

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CSCE 823

August 26, 2015

Statement of Problem

The following question appeared in the “Ask Marilyn” section of PARADE magazine in 1990 [vS91]:

> *Suppose you’re on a game show, and you’re given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what’s behind the doors, opens another door, say #3, which has a goat. He says to you, “Do you want to pick door #2?” Is it to your advantage to switch your choice of doors?*

Background Information

- 10K+ letters responded to "right" answer (most disagreed)

  You made a mistake, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.

- Perspective of question is crucial to interpretation
- Contestant: equal possibility of a car behind either door?
- Monty Hall: worse for him if contestants switch doors?
- What about the offer of a "deal" to "not switch?"
- Unclear when Monty would offer "deal" during game...

Background Information

Figure: Monty Hall's "Let's Make A Deal" TV Game Show

Previous Work

- Books on randomness, statistics, probability, etc. [Ros06] [Mlo08] [CB02]
- Novel (basis of currently running Broadway play) “Curious Incident of The Dog in The Night Time” [Had04]
- Web sites, papers, simulations refer to “a” correct answer [Cro15] [Tie91] [Tie08]
- At least one Bayesian network (BN) demo [Vom15]

Previous Work

The door containing the prize is known to Monty and thus **Prize** has impact on **Monty Opens**. Monty will never choose to open the door of your first selection so also **First Selection** has impact on **Monty Opens**. This give us the BN shown in Figure 1.

Figure 1: BN of the Monty Hall puzzle. The causal links describes that both **Prize** and **First Selection** has impact on **Monty Opens**.

Figure: Bayesian Network by Jirka Vomlel [Vom15]

Previous Work

Prize	First Selection	Monty Opens = "Door 1"	Monty Opens = "Door 2"	Monty Opens = "Door 3"
"Door 1"	"Door 1"	0	0.5	0.5
"Door 1"	"Door 2"	0	0	1
"Door 1"	"Door 3"	0	1	0
"Door 2"	"Door 1"	0	0	1
"Door 2"	"Door 2"	0.5	0	0.5
"Door 2"	"Door 3"	1	0	0
"Door 3"	"Door 1"	0	1	0
"Door 3"	"Door 2"	1	0	0
"Door 3"	"Door 3"	0.5	0.5	0

Table 3: P(Monty Opens | Prize, First Selection).

Figure: Conditional Probability Table by Jirka Vomlel [Vom15]

Comparison of Methods

- Event trees (ET) & chain event graphs (CEG) vs. BNs
- CEGs show conditional dependencies visually [BHS13]
- Decompose problem into "which strategy to use?"
- Select "maximal value" strategy for repeated contests:
  - Never Switch (No "Deal" Offered)
  - Always Switch (Never Take A "Deal")
  - Always Take A "Deal" (Fakeout Switch)
- Assume we know values V of car c and goats g_i
- What about value V of a potential "deal" d?
- "No deal" strategies assume $V(d) \ll V(c), V(g_i)$
- BNs for "no deal" strategies have same graph

Comparison of Methods

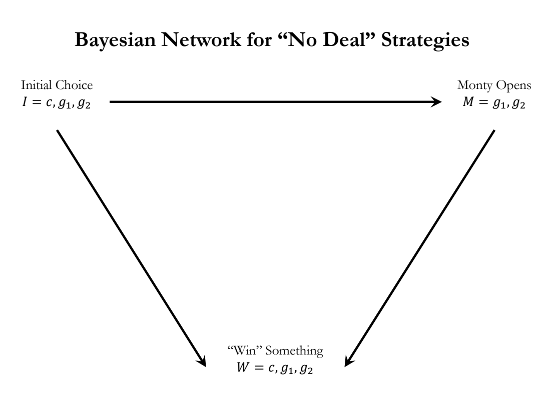

Figure: Bayesian Network for "No Deal" Strategies

Comparison of Methods

Common Dependencies for All Strategies:

$$P(I = i) = 1/3, \quad i = c, g_1, g_2$$

$$P(M = g_1 \mid I = c) = 1/2$$
$$P(M = g_2 \mid I = c) = 1/2$$

$$P(M = g_1 \mid I = g_1) = 0$$
$$P(M = g_2 \mid I = g_1) = 1$$

$$P(M = g_1 \mid I = g_2) = 1$$
$$P(M = g_2 \mid I = g_2) = 0$$

Comparison of Methods

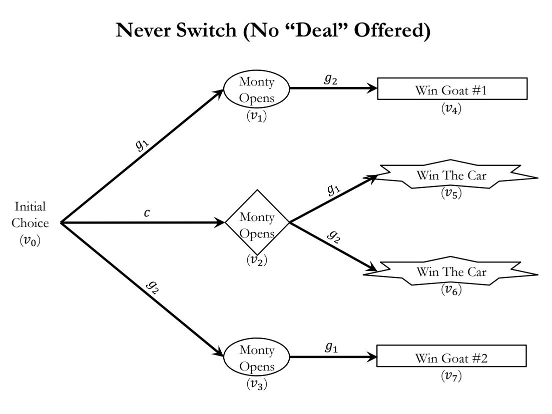

Figure: Event Tree for Never Switch (No "Deal" Offered)

Comparison of Methods

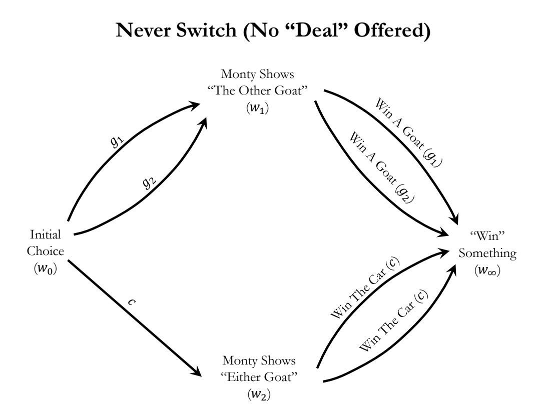

Figure: Chain Event Graph for Never Switch (No "Deal" Offered)

Comparison of Methods

Dependencies for Never Switch (No "Deal" Offered)

$$P(W = c \mid I = c, M = m) = 1, \quad m = g_1, g_2$$
$$P(W = g_1 \mid I = c, M = m) = 0, \quad m = g_1, g_2$$
$$P(W = g_2 \mid I = c, M = m) = 0, \quad m = g_1, g_2$$

$$P(W = c \mid I = g_1, M = g_2) = 0$$
$$P(W = g_1 \mid I = g_1, M = g_2) = 1$$
$$P(W = g_2 \mid I = g_1, M = g_2) = 0$$

$$P(W = c \mid I = g_2, M = g_1) = 0$$
$$P(W = g_1 \mid I = g_2, M = g_1) = 0$$
$$P(W = g_2 \mid I = g_2, M = g_1) = 1$$

Comparison of Methods

- Never Switch (No “Deal” Offered)—win the car or not?
- $P(W = c)$=1/3, $P(W = g_1)$=1/3, $P(W = g_2)$=1/3
- Assume we know values V for goats (g_i) and car (c)
- Never switch (or take a “deal”) if $V(g_i) > 2V(c)$

Comparison of Methods

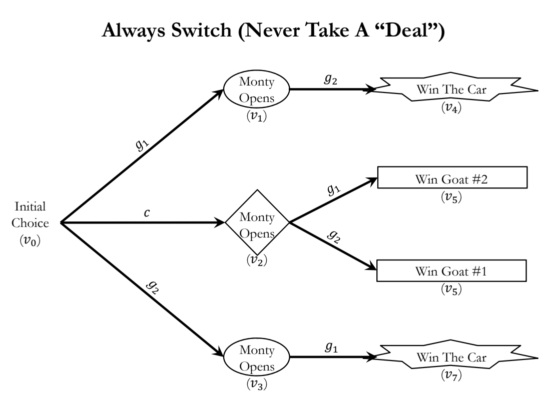

Figure: Event Tree for Always Switch (Never Take A "Deal")

Comparison of Methods

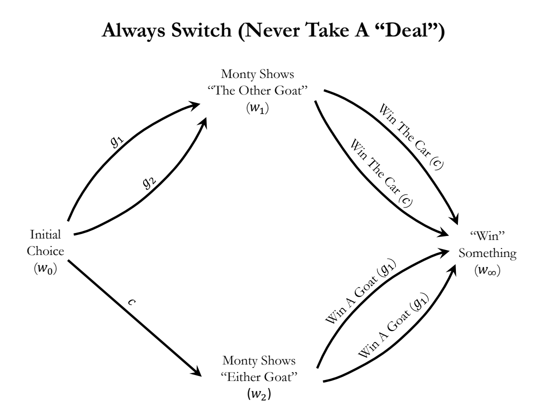

Figure: Chain Event Graph for Always Switch (Never Take A "Deal")

Comparison of Methods

Dependencies for Always Switch (Never Take A “Deal”)

$$
\begin{aligned}
P(W &= c &| \; I = c, M = m) &= 0, & m = g_1, g_2 \\
P(W &= g_1 &| \; I = c, M = m) &= 1/2, & m = g_1, g_2 \\
P(W &= g_2 &| \; I = c, M = m) &= 1/2, & m = g_1, g_2
\end{aligned}
$$

$$
\begin{aligned}
P(W &= c &| \; I = g_1, M = g_2) &= 1 \\
P(W &= g_1 &| \; I = g_1, M = g_2) &= 0 \\
P(W &= g_2 &| \; I = g_1, M = g_2) &= 0
\end{aligned}
$$

$$
\begin{aligned}
P(W &= c &| \; I = g_2, M = g_1) &= 1 \\
P(W &= g_1 &| \; I = g_2, M = g_1) &= 0 \\
P(W &= g_2 &| \; I = g_2, M = g_1) &= 0
\end{aligned}
$$

Comparison of Methods

- Always Switch (Never Take A "Deal")—win the car or not?
- $P(W = c)$=2/3, $P(W = g_1)$=1/6, $P(W = g_2)$=1/6
- Normalized relative frequency of events w/ $P \neq 0$
- Always switch (never take a "deal") if $V(g_i) < 2V(c)$
- What if $V(d) > 2V(c) \gg V(g_i)$?
- Denote $c + d$ as c^+, $g_i + d$ as g_i^+

Comparison of Methods

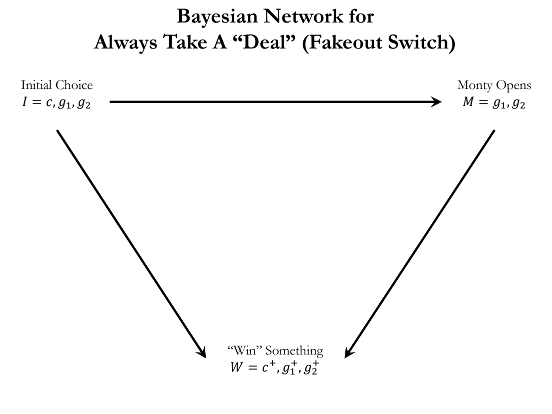

Figure: Bayesian Network for Always Take A "Deal" (Fakeout Switch)

Comparison of Methods

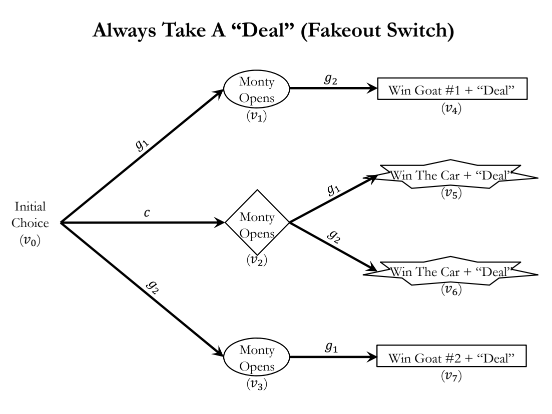

Figure: Event Tree for Always Take A "Deal" (Fakeout Switch)

Comparison of Methods

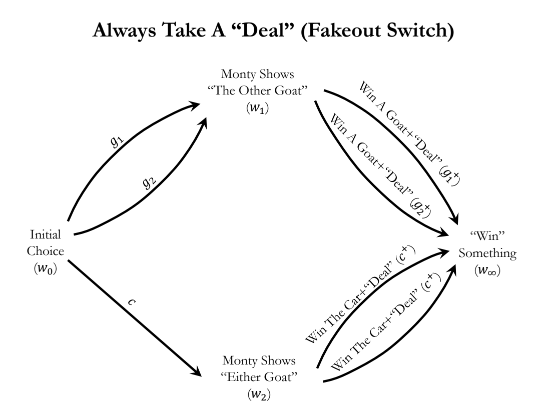

Figure: Chain Event Graph for Always Take A "Deal" (Fakeout Switch)

Comparison of Methods

Dependencies for Always Take A "Deal" (Fakeout Switch)

$$P(W = c^+ \mid I = c, M = m) = 1, \quad m = g_1, g_2$$
$$P(W = g_1^+ \mid I = c, M = m) = 0, \quad m = g_1, g_2$$
$$P(W = g_2^+ \mid I = c, M = m) = 0, \quad m = g_1, g_2$$

$$P(W = c^+ \mid I = g_1, M = g_2) = 0$$
$$P(W = g_1^+ \mid I = g_1, M = g_2) = 1$$
$$P(W = g_2^+ \mid I = g_1, M = g_2) = 0$$

$$P(W = c^+ \mid I = g_2, M = g_1) = 0$$
$$P(W = g_1^+ \mid I = g_2, M = g_1) = 0$$
$$P(W = g_2^+ \mid I = g_2, M = g_1) = 1$$

Comparison of Methods

- Always Take A “Deal” (Fakeout Switch)—win car + “deal?”
- $P(W = c^+)$=1/3, $P(W = g_1^+)$=1/3, $P(W = g_2^+)$=1/3
- Always take a deal (fakeout switch) if $V(g_i^+) > 2V(c^+)$
- This only works if $V(g_i) \ll 2V(c) < V(d)$

Conclusions and Limitations

- CEGs can help refine or improve BNs [BHS13]
- CEGs helped to write down probability tables for BNs
- Visually compared strategies by looking at CEGs
- Multiple limitations to the analysis:
  - Monty will not always follow our "rules" [Ell95]
  - "Let's Make A Deal" goes on even if it keeps losing money?
  - Repeated tries by a team of united contestants?
- Monty wants contestants to go with "intuitive" answer

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